MCS 245-Differential Equations　　Fall 2014　　Midterm #1

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Please show as much work as possible to get full or partial credit. Please circle your answers.

1. **(12 pts.)** Classify the following differential equations depending on their type, order and linearity. In case of non-linearity explain why.

a) $y\frac{dy}{dx} = x.$

b) $\frac{d^2y}{dx^2} - 16y = 0.$

c) $\left(\frac{dy}{dx}\right)^3 = y^2.$

d) $\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) = xyu.$

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2. **(15 pts.)** Determine whether the *Existence and Uniqueness Theorem for First order IVP's* implies that there is a unique solution for the below IVP. Why or why not? Show your work.

$$\frac{dy}{dx} = 3x - \sqrt[3]{y-1}, \qquad y(2) = 1$$

3. (13 pts.) Find the solution of the following IVP,

$$\frac{dy}{dx} - x^2y^2 = x^2, \quad y(0) = \frac{\pi}{4}.$$

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4. (20 pts.) Find the general solution of, $y' = \frac{y(y+2x)}{3x^2}$.

5. (20 pts.) Solve the *Bernoulli* equation, $y' + \frac{1}{2(x+1)}y = (1 - x^2)y^3$.

6. **(20 points)** Find m, n such that $\mu = x^m y^n$ is an integrating factor for $y^2 dx + x(x + y)dy = 0$, and then solve it.

Extra Credit Problem: (10 points)

Find the general solution of the following differential equations.

a) $9y''(t) + 12y'(t) + 4y(t) = 0.$

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b) $4z''(x) - 4z'(x) + 17z(x) = 0.$