MCS 245-Differential Equations Fall 2013 Midterm #1
Instructor: B. Kaymakçalan Name:________________________

Please show as much work as possible to get full or partial credit. Please circle your answers.

1. **(8 pts.)** For the equation $8\frac{d^4y}{dx^4} = x(1-x)$ answer the following

a) Is this ordinary or partial differential equation (DE)?

b) What is the order of this DE?

c) What is the dependent variable and what is the independent variable?

d) Is this a linear or nonlinear equation?

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2. **(12 pts.)** Classify the following equations as separable, linear, exact, homogeneous (v=y/x type) or none of them. Note that some equations may have more than one classification. Justify.

a) $y^2dx + (2xy + \cos y)dy = 0$

b) $yxdx + dy = 0$

c) $(ye^{-2x} + y^3)dx - e^{-2x}dy = 0$

3. (12 pts.) Determine whether the *Existence and Uniqueness Theorem for First order IVP's* implies that there is a unique solution for the below IVP. Why or why not? Show your work.

$$\frac{dy}{dx} = 3x - \sqrt[3]{y-2}, \qquad y(1) = 2$$

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4. (8 pts.) Determine if the relation $x^2 - \sin(x+y) = 1$ is the implicit solution to the equation

$$\frac{dy}{dx} = 2x\sec(x+y) - 1.$$

5. (14 pts.) Find the solution to the equation $4y''-4y'+5y=0, \quad y(0)=1, \quad y'(0)=-11/2$.

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6. (22 pts.) Solve the *Bernoulli* equation $\frac{dy}{dx}+\frac{y}{x-2}=5(x-2)\sqrt{y}$

7. **(24 pts.)** Find the general solution to the equation $y'' = 5x^{-1}y' - 13x^{-2}y, \quad x > 0$.
(Hint: Can you recognize the type of this equation?)

Extra Credit Problem: (10 points)

Show that the equation $(2x^2 + y)dx + (x^2 y - x)dy = 0$ is not exact. Find an integrating factor that would make this equation exact and verify that the new equation is exact and then solve the new equation.